Compositionality and Set Theory

Semantics and Compositionality

What is Semantics?

The science of **meaning**

What is Semantics? (pt 2)

Seeks to answer questions such as:

Why does a word mean what it does?

How are we able to understand the meanings of indefinitely many sentences?

How do different meanings interact with each other in the context of phrases and sentences?

What is Compositionality?

The meanings of larger expressions are composed by the meanings of smaller expressions within them.

> The meaning of the sentence ‘Noah walked Ellie’ is composed out of the meanings of ‘Noah’, ‘walked’, and ‘Ellie’

What is Compositionality? (pt 2)

The meanings of larger expressions are determined not just by what the meanings of the smaller expressions are, but also how they are arranged within the larger expressions.

‘Noah walked Ellie’ ≠ ‘Ellie walked Noah’

Compositional Rules for English

The rules that determine how the smaller meanings contribute to the bigger meaning are not always straightforward or obvious.

Compare the relation between the meanings of ‘George’ and ‘leave’ in (A) and (B):

(A) George is eager to leave.

≈ George is eager for himself to leave.

≉ George is eager for people to leave him.

(B) George is easy to leave.

≉ It is easy for George to leave people.

≈ It is easy for people to leave George.

(A) says something about George leaving, while (B) says something about leaving George.

Compositional Semantics with Set Theory

Modelling Meaning with Sets: Predicates

We can model the meaning of predicates with sets. The meaning of a predicate can be modelled by the set of all things that satisfy that predicate.

The predicate ‘is a dog’ is modelled by: {x | x is a dog}.

The predicate ‘has been eaten by a wolf’ is modelled by: {x | x has been eaten by a wolf}.

Modelling Meaning with Sets: Predication

We can now model predication (i.e. when a predicate is applied to a subject) with set membership.

When predicate P is applied to subject S, the resulting expression is true just in case $S \in \{x \mid x \text{ is } P\}$

‘Ellie is a dog’ is true just in case $\text{Ellie} \in \{x \mid x \text{ is a dog}\}$

‘Noah is not a dog’ is true just in case $\text{Noah} \notin \{x \mid x \text{ is a dog}\}$

Modelling Meaning with Sets: Logical Vocabulary

The meanings of logical words like ‘all’, ‘every’, ‘some’, and ‘most’ can be modelled using set-theoretic relations, operations, and cardinalities.

All A’s are B’s ≈ Every A is a B

‘All dogs are mammals’ ≈ ‘Every dog is a mammal’

How can we model a sentence of one of the above forms using set theory? (Hint: You will need to use the subset relation.)

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‘All dogs are mammals’ ≈ ‘Every dog is a mammal’

How can we model the sentences above using set theory? In other words, give an expression in the language of set theory that is true if and only if ‘All dogs are mammals’ is true. (Hint: You will need to use the subset relation.)

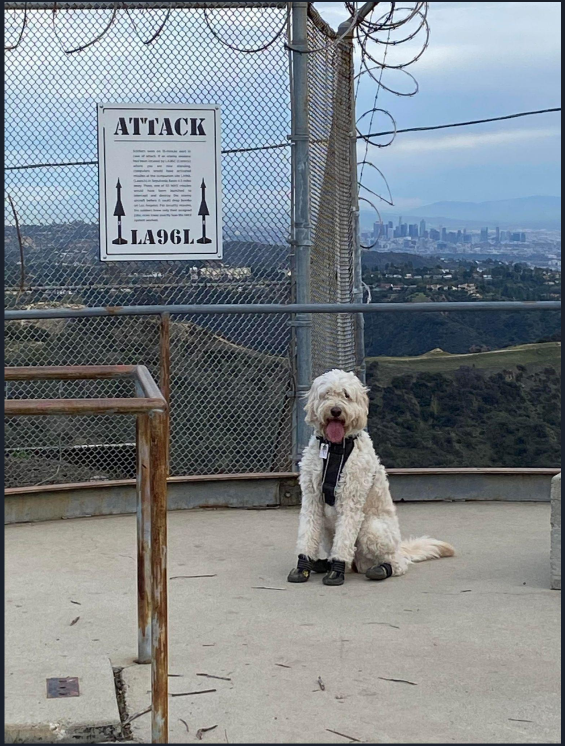
$\{x \mid x \text{ is a dog}\} \subseteq \{x \mid x \text{ is a mammal}\}$

Modelling Meaning with Sets: ‘Some’

Some A’s are B’s

Some dogs wear boots

How can we model this in set-theoretic language? (Hint: use intersection and the null set.)



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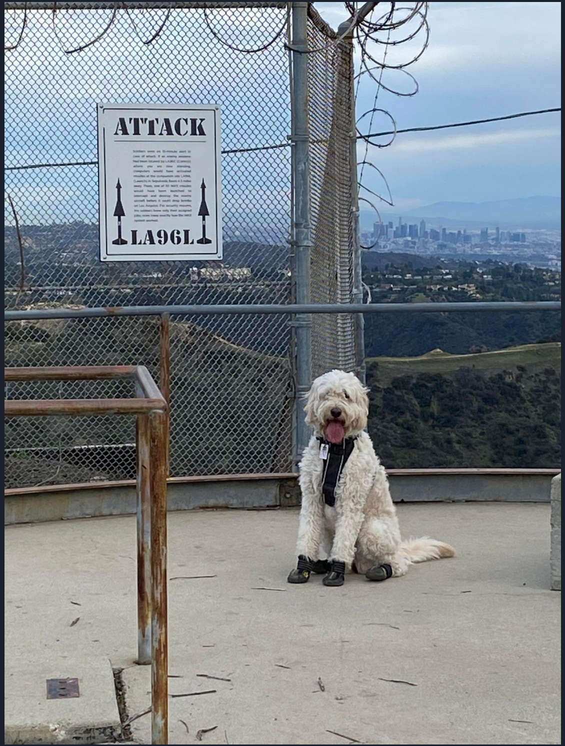
$\{x \mid x \text{ is a dog}\} \cap \{x \mid x \text{ wears boots}\} \neq \varnothing$



Modelling Meaning with Sets: ‘Most’

Most A’s are B’s

Most dogs have fur

How can we model this in set-theoretic language? (Hint: use intersection and arithmetic comparisons between cardinalities.)

Modelling Meaning with Sets: ‘Most’

$$|\{x \mid x \text{ is a dog}\} \cap \{x \mid x \text{ has fur}\}| > |\{x \mid x \text{ is a dog}\} \cap \{x \mid x \text{ does not have fur}\}|$$

Modelling Meaning with Sets: ‘And’

‘And’ can be used to join two predicate phrases together to form a complex predicate phrase.

S is A and B.

Ellie is dirty and has a ball.

How can we model this in set-theoretic language? (Hint: use membership and intersection.)

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Ellie is dirty and has a ball.

How can we model this in set-theoretic language? (Hint: use membership and intersection.)

Ellie $\in$ {x | x is dirty} $\cap$ {x | x has a ball}

Modelling Meaning with Sets: ‘Or’

‘Or’ can also be used to join two predicate phrases together to form a complex predicate phrase.

S is A or B

Ellie is comfortable or upset.

How can we model this in set-theoretic language? (Hint: use membership and union.)

Modelling Meaning with Sets: ‘Or’

‘Or’ can also be used to join two predicate phrases together to form a complex predicate phrase.

S is A or B

Ellie is comfortable or upset.

How can we model this in set-theoretic language? (Hint: use membership and union.)

Ellie $\in$ {x | x is comfortable} $\cup$ {x | x is upset}